It is well known that spontaneous emission, typically assumed to be an independent process for each atom, can be correlated due to the interference of light emitted by different atoms. Over the past five decades, cooperative radiation phenomena such as Dicke's superradiance has been explored in systems ranging from individual atoms to black holes. Recently, such cooperative radiation emerged as a promising method for manipulating light and matter in systems ranging from unordered gases to ordered atomic arrays to two-dimensional semiconductor materials. For instance, cooperative effects can lead to entire manifolds of protected subradiant states whose decay rates approach zero with increasing system size, and to the realization of atomically thin mirrors with exotic properties. Alternatively, controlling the response of the atomic medium by combining spontaneous emission with strong, long-range could be used to control entangled many-body states of atoms. I will discuss several theoretical ideas, ongoing experiments to implement them, as well as potential applications of such effects, including realization of topological optical systems, quantum nonlinear optics, molecule cooling, and quantum information processing.