Our existing physical laws are unable to explain several features of the observed Universe. The nature of the dark matter that holds individual galaxies together and of the dark energy that drives different galaxies away from each other both require new physics beyond the Standard Model and general relativity. The preponderance of matter over antimatter likewise requires some new baryon-number violation beyond that in the Standard Model. Explanations for the primordial density inhomogeneities observed in the cosmic microwave background all involve new physics. I will review these questions, discuss some existing avenues to make progress, emphasizing several ways in which considerations of symmetry and geometry may play a role in the quest for new cosmological physics.