Theory predicts a variety of peculiar quantum-mechanical phases for condensed matter systems at zero temperature, with surprising features. In some phases, extra degrees of freedom with extremely low energy appear when localized “defects” are introduced. In the simplest class of these phases, when there are \( N \) defects present, the number of states in the low energy Hilbert space is of order \( 2^{N/2} \), in contrast to \( 2^N \), which would be the case if there were a conventional two-level system associated with each defect. Instead, each defect is accompanied by a localized zero-energy state for a “Majorana fermion”, which is, in turn, a realization in condensed matter systems of an exotic concept introduced originally in elementary particle physics. Moreover, the localized defects exhibit the property of “non-Abelian statistics”: if defects can be moved around each other, or if two identical defects can be interchanged, one can produce unitary transformations in the low-energy Hilbert space, which can depend on the order in which operations are performed.

In my talk, I will try to explain these various concepts and show how they may be realized in condensed matter systems.